Biscovey
Calculation Policy

The National Curriculum 2014

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics and a sense of enjoyment and curiosity about the subject.

The aims of this policy

Mastery in mathematics is for all, and the aim of this policy is to ensure all children leave our schools with a secure understanding of the four operations and can confidently use and apply both written and mental calculation strategies in a range of contexts. It aims to ensure consistent strategies, models and images are used across the schools to embed and deepen children's learning and understanding of mathematical concepts.

How should this policy be used?

This policy has been designed to support the teaching and planning of mathematics in our schools. The policy only details the strategies, and teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum. The examples and illustrations are not exhaustive but provide and overall picture of what the mathematics in our school should look like. This is not a scheme of work and should be used in conjunction with national curriculum documents and the White Rose Scheme of learning used in our schools.

This policy sets out the progression of strategies and written methods which children will be taught as they develop in their understanding of the four operations. Strategies are set out in a Concrete, Pictorial, Abstract (CPA) approach to develop children's deep understanding and mastery of mathematical concepts. Children use concrete objects to help them make sense of the concept or problem; this could be anything from real or plastic fruit, to straws, counters or cubes. This is then developed through the use of images, models and children's own pictorial representations before moving on to the abstract mathematics. Children will travel along this continuum again and again, often revisiting previous stages when a concept is extended or a new one is taught.

Although the strategies are taught in a progressive sequence, they are designed to equip children with a 'toolbox' of skills and strategies that they can apply to solve problems in a range of contexts. So as a new strategy is taught it does not necessarily supersede the previous, but builds on prior learning to enable children to have a variety of strategies to select from. As children become increasingly independent, they will be able to and must be encouraged to select those strategies which are most efficient for the task.

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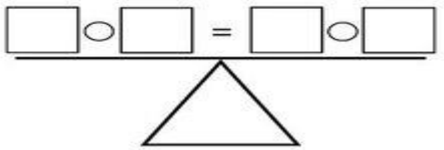
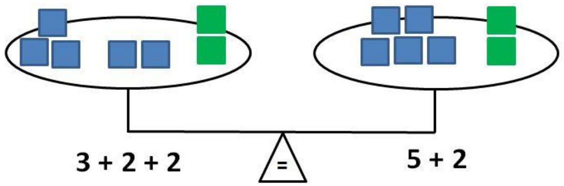
The strategies are separated into the 4 operations for ease of reference. However, it is intended that addition and subtraction, and multiplication and division will be taught together to ensure that children are making connections and seeing relationships in their mathematics. Therefore, some strategies may be taught simultaneously, for example, counting on (addition) and counting back (subtraction).

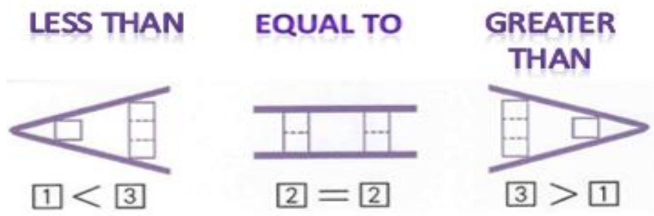
Children should be moved through the strategies at a pace appropriate to their age related expectations as defined in the EYFS Development Matters and 2014 National Curriculum for Key Stage 1 and Key Stage 2. Effective teaching of the strategies relies on increasing levels of number sense, fluency and ability to reason mathematically. Children must be supported to gain depth of understanding within the strategy through the CPA approach and not learn strategies as a procedure without conceptual understanding.

Teaching equality

It is important that when teaching the 4 operations that equality (=) is also taught appropriately. Misconceptions that = means that children must 'do something' and that it indicates that an answer is required are common and must be addressed early on. Teachers should present children with number sentences and problems which place the = sign in different positions, different context and include missing box problems. In the concrete phase scales and Numicon provide a useful resource to demonstrate equality. Pictorial representations of equality can be used as shown below:







Importance of vocabulary

The 2014 National Curriculum places great emphasis on the importance of pupils using the correct mathematical language as a central part of their learning. Children will be unable to articulate their mathematical reasoning if they lack the mathematical vocabulary required to do so. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary indicated for each of the 4 operations. High expectations of the mathematical language used are essential, with teachers modelling and only accepting what is correct or which has been agreed as the consistent approach within our schools. For example:

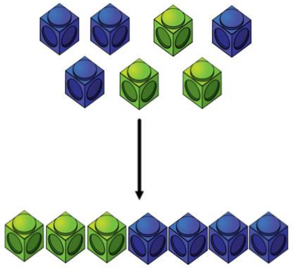
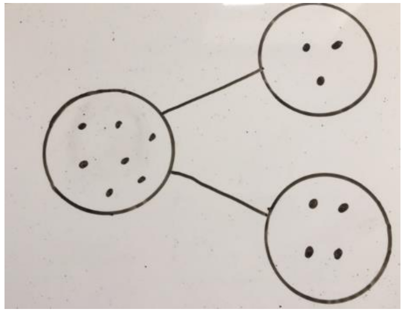
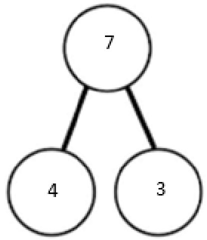
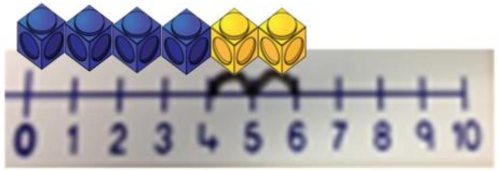
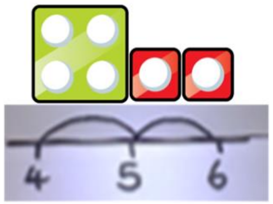
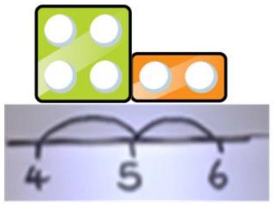
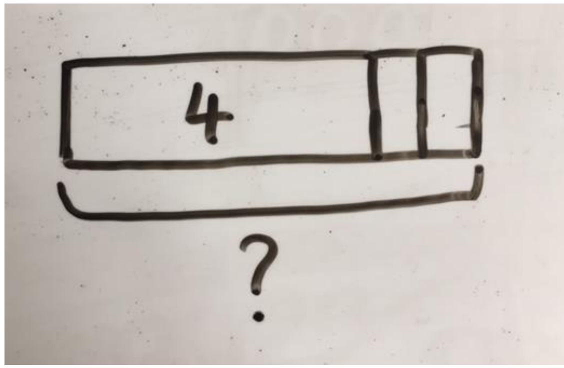
✓	×
ones	units
is equal to	equals
zero	oh (the letter O)
number sentences	sums

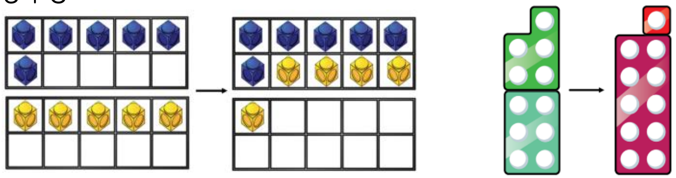
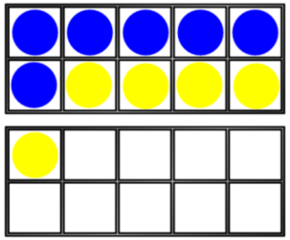
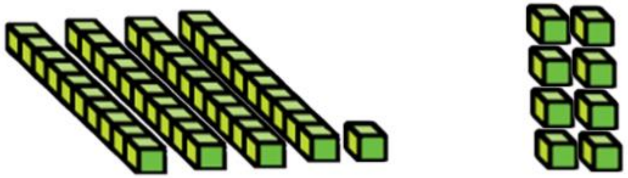
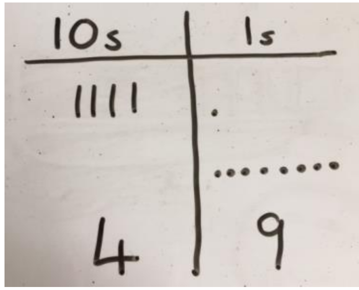
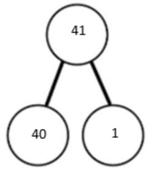
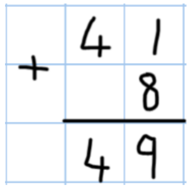
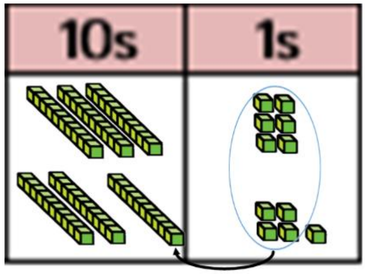
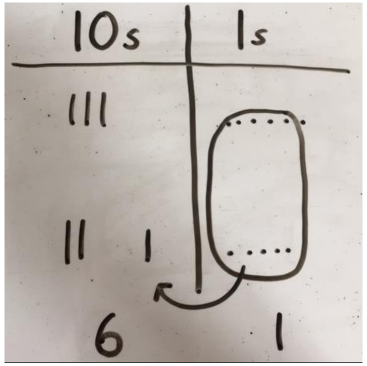
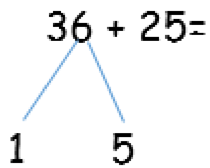
The principle focus of mathematics within each stage.

EYFS	Key Stage 1	Lower Key Stage 2	Upper Key Stage 2
The principle focus of learning in the early years is to ensure that practitioners teach children by ensuring challenging, playful opportunities across the prime and specific areas of learning and development. They foster the characteristics of effective early learning - Playing and exploring; Active learning; Creating and thinking critically. In mathematics this involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measure By the end of reception, children should be able to count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.	The principal focus of mathematics teaching is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money. By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency. Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.	The principal focus of mathematics teaching is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers. At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should ensure that they can use measuring instruments with accuracy and make connections between measure and number. By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work. Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.	The principal focus of mathematics teaching is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio. At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages. Pupils should read, spell and pronounce mathematical vocabulary correctly.

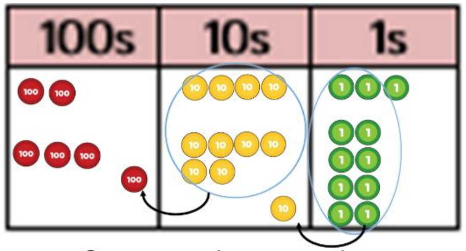
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, ‘is equal to’ ‘is the same as’.

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.   	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

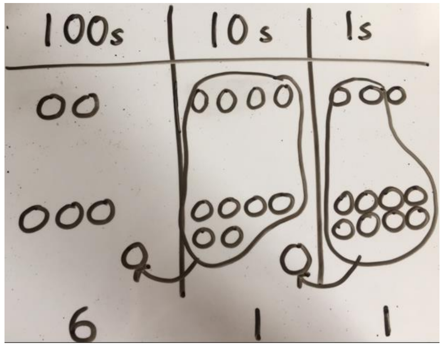
Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25  6 1	Chidlren to represent the base 10 in a place value chart. 	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61 1

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



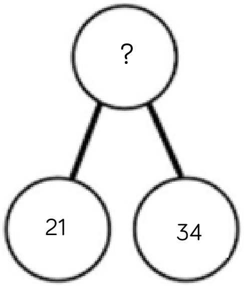
6 1 1

Chidren to represent the counters in a place value chart, circling when they make an exchange.




```
  243
+368
  611
  1 1
```

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

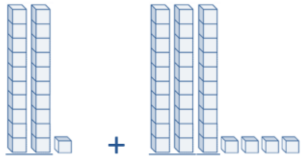
21 + 34 = 55. Prove it

```
  21
+34
____
```

21 + 34 =

☐ = 21 + 34

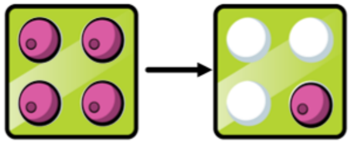
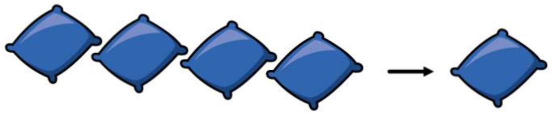
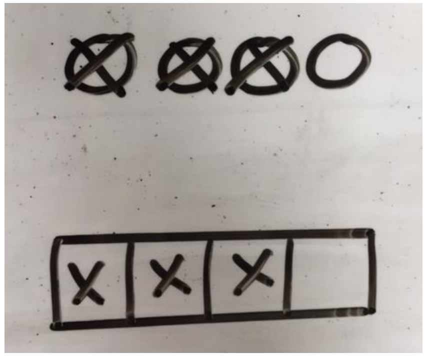
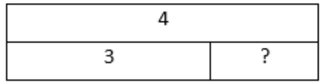
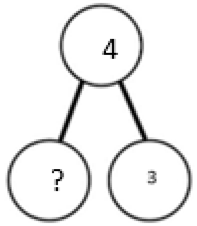
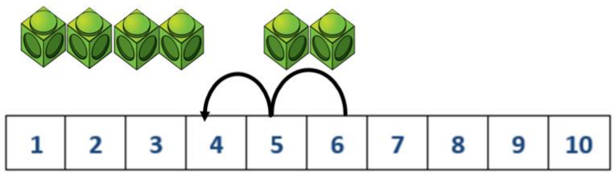
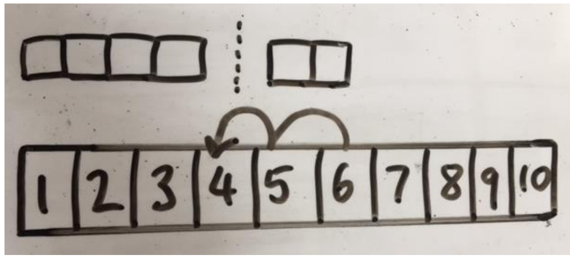
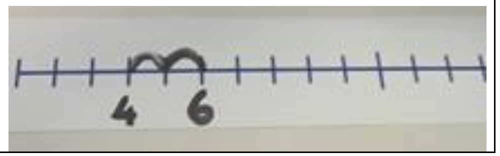
Calculate the sum of twenty-one and thirty-four.



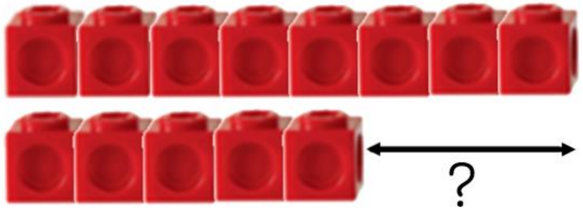
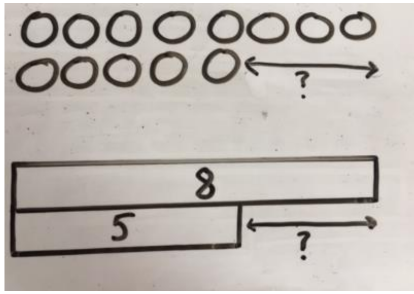
Missing digit problems:

Calculation policy: Subtraction

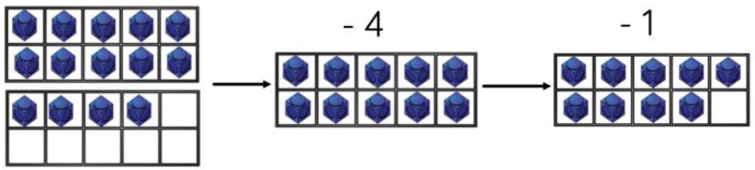
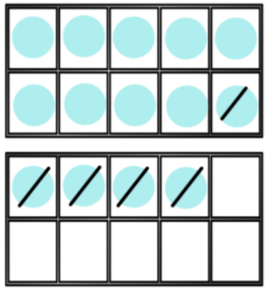
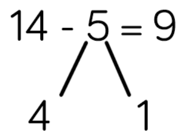
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

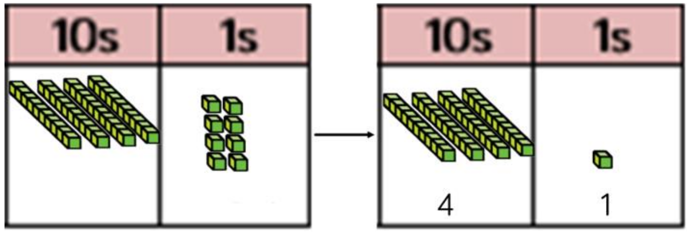
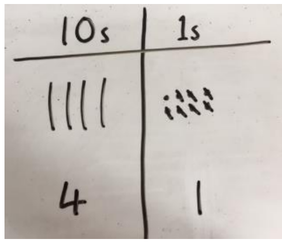
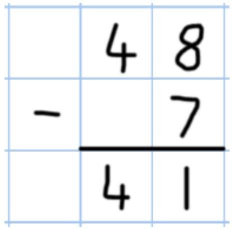
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1  	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = ☐ = 4 – 3  
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4  	Children to represent what they see pictorially e.g.  	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line  

<table>
<tr>
<td>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.

</td>
<td>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.

</td>
<td>Find the difference between 8 and 5.

8 – 5, the difference is ☐

Children to explore why
9 - 6 = 8 – 5 = 7 – 4 have the same difference.</td>
</tr>
<tr>
<td>Making 10 using ten frames.
14 – 5

</td>
<td>Children to present the ten frame pictorially and discuss what they did to make 10.
</td>
<td>Children to show how they can make 10 by partitioning the subtrahend.


14 – 4 = 10
10 – 1 = 9</td>
</tr>
<tr>
<td>Column method using base 10.
48-7

</td>
<td>Children to represent the base 10 pictorially.

</td>
<td>Column method or children could count back 7.

</td>
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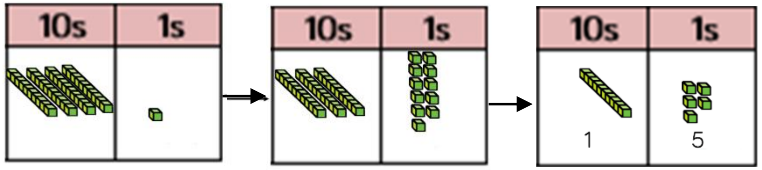
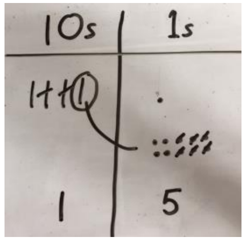
</table>

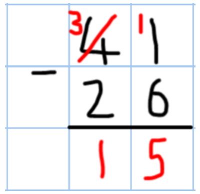
Column method using base 10 and having to exchange.
41 – 26



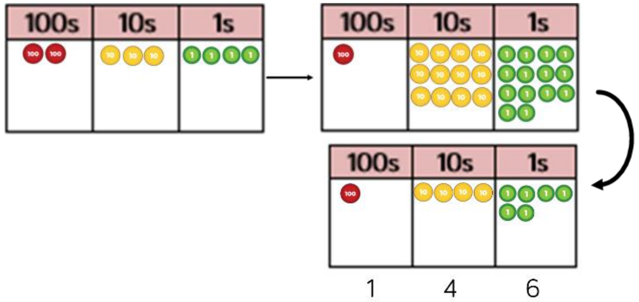
Represent the base 10 pictorially, remembering to show the exchange.



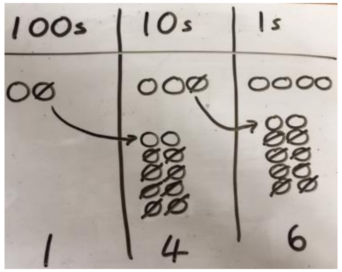
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.



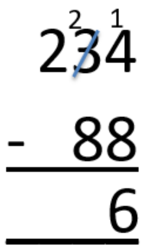
Column method using place value counters.
234 – 88



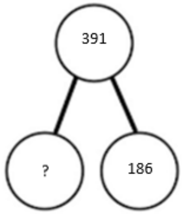
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal colum method. Children must understand what has happened when they have crossed out digits.



Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

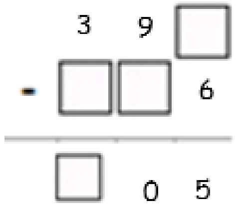
Calculate the difference between 391 and 186.

☐ = 391 – 186

391
-186

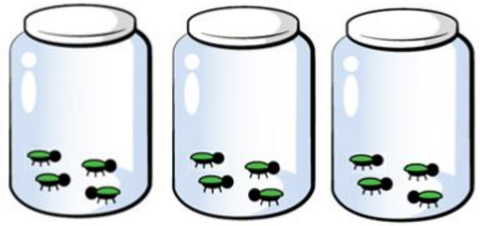
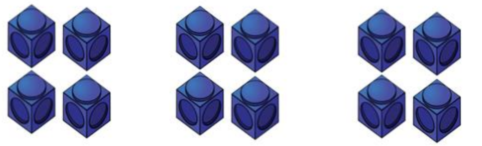
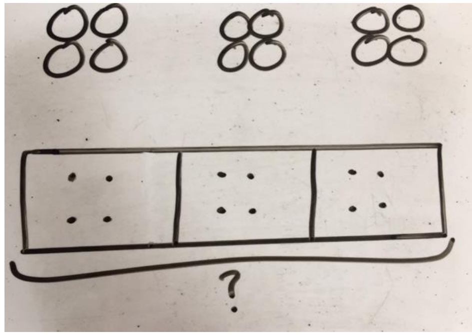
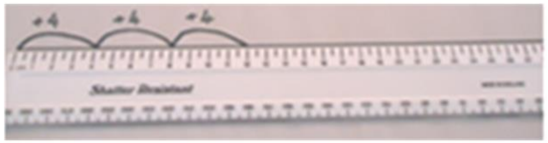
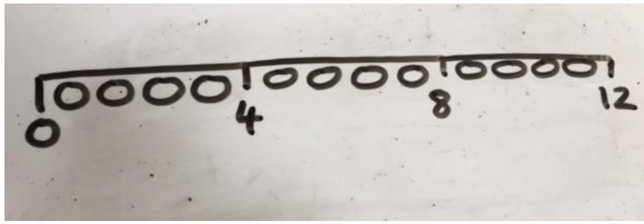
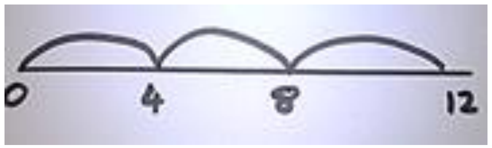
What is 186 less than 391?

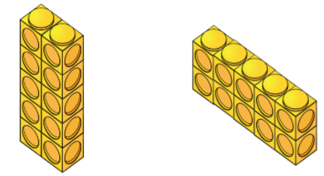
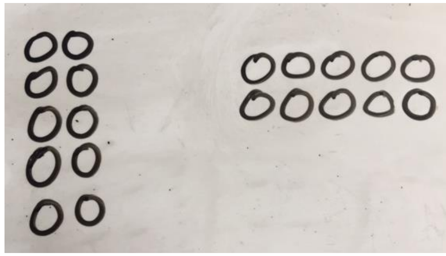
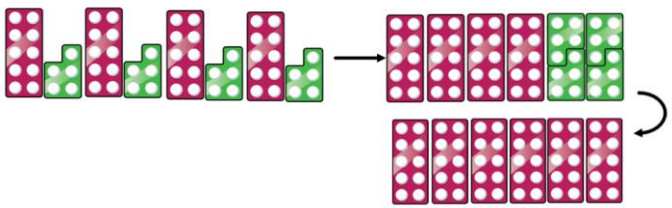
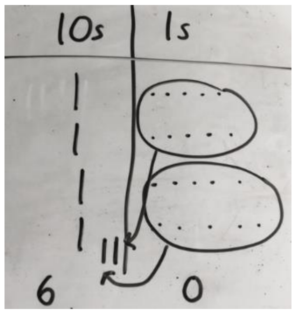
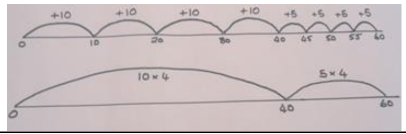
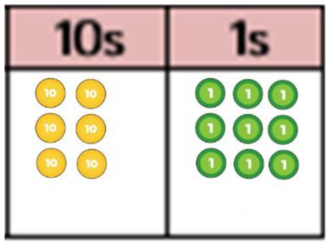
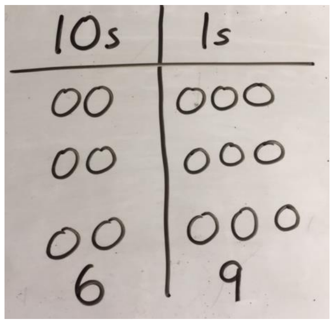
Missing digit calculations



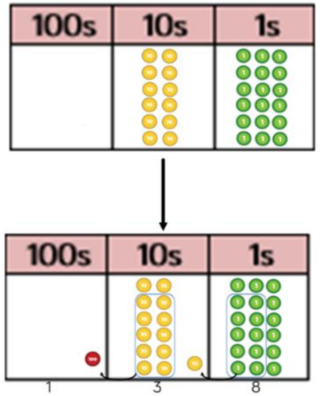
Calculation policy: Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

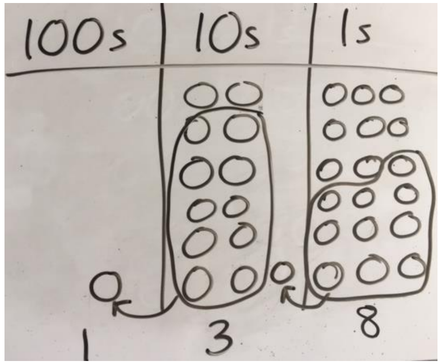
Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group.  	Children to represent the practical resources in a picture and use a bar model.  	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.:  	Abstract number line showing three jumps of four. 3 × 4 = 12 

Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4×15 10 5 $10 \times 4 = 40$ $5 \times 4 = 20$ $40 + 20 = 60$ A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3×23  6 9	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3×23 $3 \times 20 = 60$ $3 \times 3 = 9$ 20 3 $60 + 9 = 69$ $$\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \\ \hline \end{array}$$

Formal column method with place value counters.
6 x 23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

6 x 23 =

```
   23
×   6
  138
  1 1
```

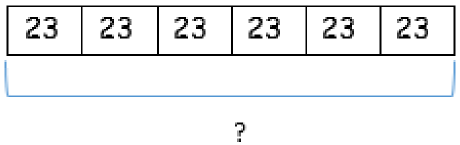
When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

```
     1  2  4
×       2  6
     7  4  4
     1  2
  2  4  8  0
  3  2  2  4
  1  1
```

Answer: 3224

Conceptual variation; different ways to ask children to solve 6 × 23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

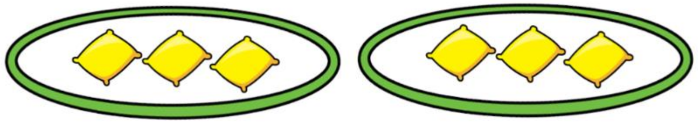
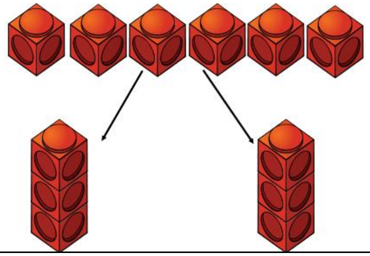
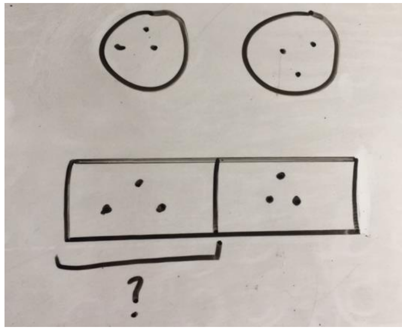
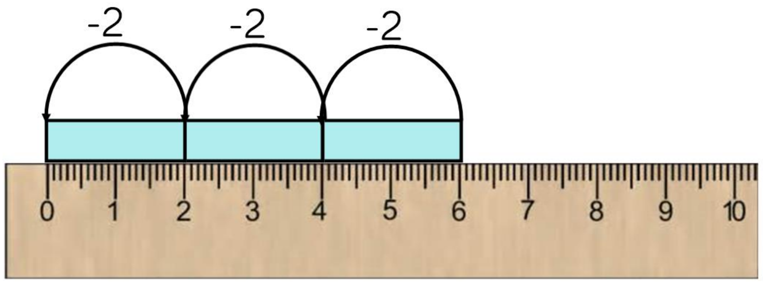
☐ = 6 × 23

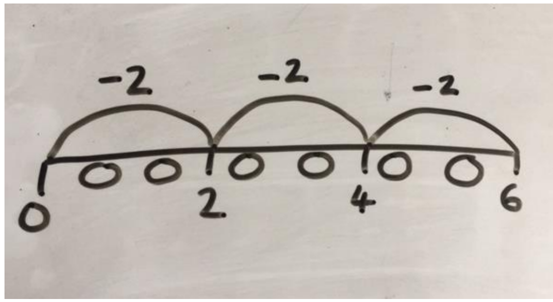
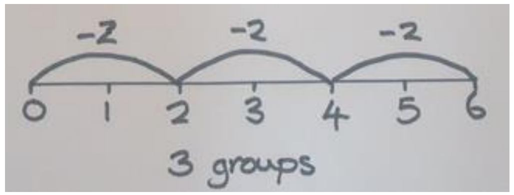
```
   6        23
× 23      ×  6
____      ____
```

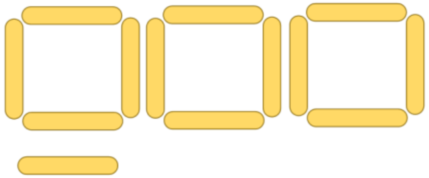
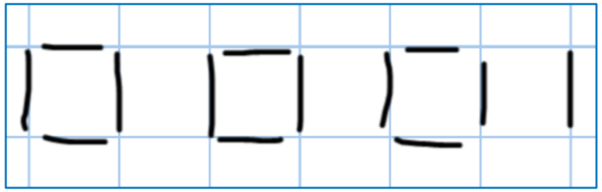
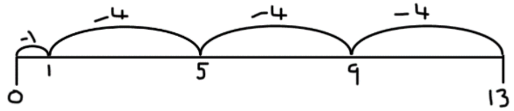
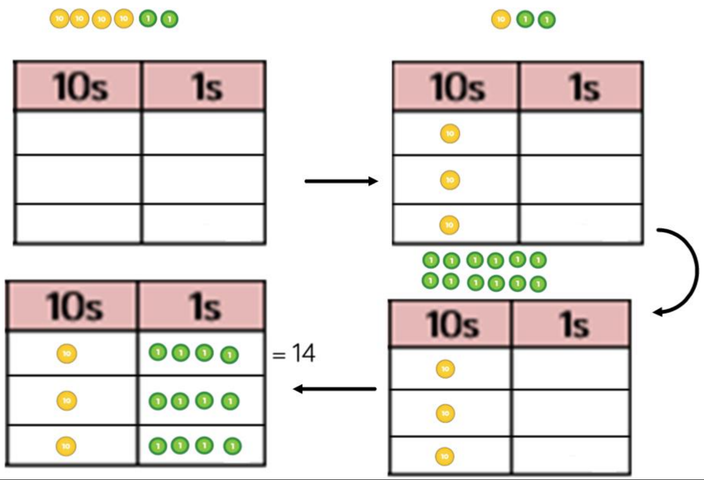
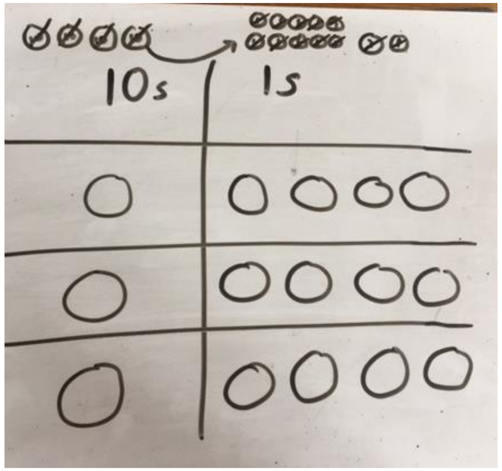
What is the calculation?
What is the product?

Calculation policy: Division

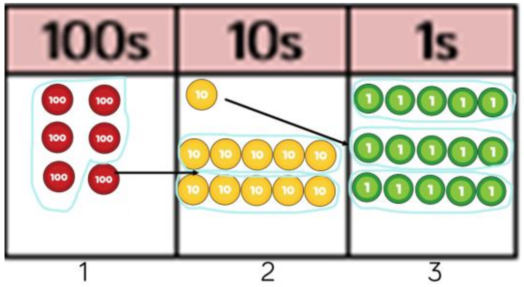
Key language: share, group, divide, divided by, half.

<table>
<tr><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Sharing using a range of objects.
6 ÷ 2

</td>
<td>Represent the sharing pictorially.

</td>
<td>6 ÷ 2 = 3
<table><tr><td>3</td><td>3</td></tr></table>
Children should also be encouraged to use their 2 times tables facts.</td>
</tr>
<tr>
<td>Repeated subtraction using Cuisenaire rods above a ruler.
6 ÷ 2


3 groups of 2</td>
<td>Children to represent repeated subtraction pictorially.

</td>
<td>Abstract number line to represent the equal groups that have been subtracted.

</td>
</tr>
</table>

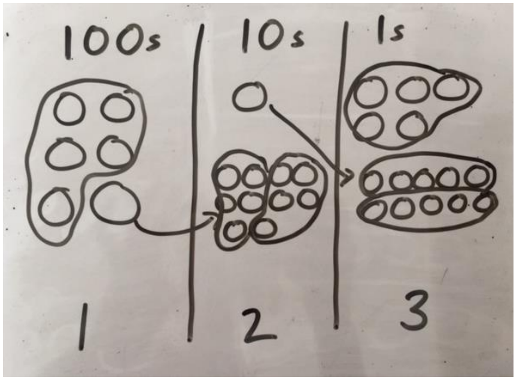
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 – 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Short division using place value counters to group.
615 ÷ 5

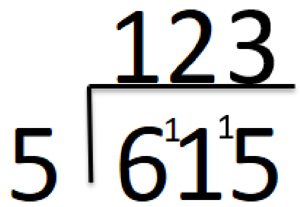


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

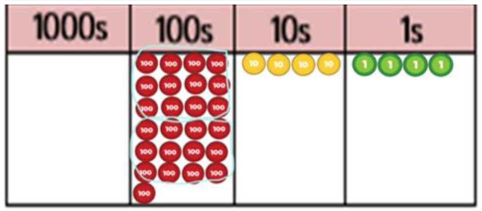


Children to the calculation using the short division scaffold.

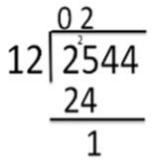


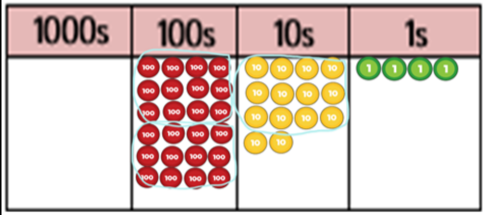
Long division using place value counters
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



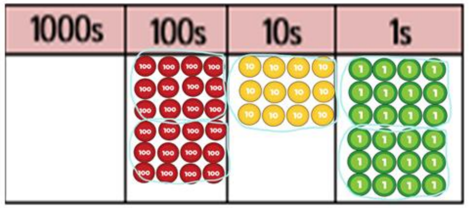
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.





After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

```
    0 2 1
12 |2544
    24
     14
     12
      2
```

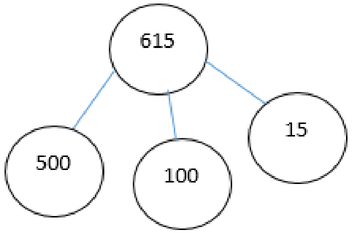


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

```
    0 2 1 2
12 |2544
    24
     14
     12
      24
      24
       0
```

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

5 ⟌615

615 ÷ 5 =

☐ = 615 ÷ 5

What is the calculation?
What is the answer?